Radiometric force in dusty plasmas

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It is shown that inhomogeneous heating of the dust grain by ion flow in a glow discharge results in a photophoresislike force. According to our estimations, this radiometric force can be comparably-valued with the ion-drag force under the conditions of microgravity dusty plasma experiments.

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Numerous industrial applications, such as material proceeding, has triggered active research on the phenomena associated with dust dynamics in a low-pressure glow discharge. A dusty plasma [1,2] is formed by introducing micron-sized grains in a plasma. Since the grains are negatively charged due to the higher mobility of electrons with respect to ions, the medium is composed of particles with fixed charge (electrons and ions) and variable charge (dust grains). Typically, the grains can be easily charged to 10^4-10^5 electron charges. In a low-temperature radiofrequency discharge, the grains can be electrically suspended in the sheath above the electrodes [3], where the gravity is exactly balanced by the electric force. Under certain conditions, the grains form ordered lattice structures, known as Coulomb crystals.

The plasma boundary near the electrodes is characterized by highly nonequilibrium conditions. In particular, the grains undergo a supersonic (according to the Bohm criterion) ion flow, resulting both in an ion-drag force and specific attractive forces between the grains [4]. It is well-known from aerosol physics that there are various forces provided by the density, temperature, etc. gradients of the neutral gas. However, aside from thermophoresis [5,6], the role of such forces in a dusty plasma is an open question.

In the present paper we focus on the so-called radiometric force. We found that the ion flow results in inhomogeneous temperature distribution of the grain surface. Then the interaction with the neutral gas results in a force similar to photophoresis [7], but it is provided by a plasma recombination at the grain surface.

Let Q be the energy released in each act of ion recombination. Typically, the value of Q is of the order of ten eV. We assume that this energy is absorbed by the grain and results in the heating of its surface. Recall that the ion flow in the sheath near the electrodes is strongly nonisotropic. The widely spread approximation is that zero-temperature ions move towards the electrode with the same supersonic velocity Mc_s, where $M>1$ is Mach number, and c_s is the ion sound velocity. Then the energy flux per a unit square of the grain surface oriented normally to the ion velocity, can be estimated as

$$J_0=\alpha n_i M c_s Q, \tag{1}$$

where n_i is the ion number density. The dimensionless factor α, which is typically of the order of two, takes into account the attraction of ions by the negatively-charged grain.

The steady energy flux due to the plasma recombination affects only one side of the grain resulting in some inhomogeneous heating. The stationary temperature distribution $T(\mathbf{r})$ inside the grain can be evaluated by means of the Fourier's equation

$$\operatorname{div}[\varkappa \operatorname{grad} T(\mathbf{r})]=0, \tag{2}$$

where $\varkappa$ is the grain thermal conductivity. Equation (2) should be supplemented with a boundary condition. We characterize the grain surface by a unit vector $\mathbf{n}$ directed outwards. Then the boundary condition reads

$$J_s=\varkappa\left(\frac{\partial T}{\partial \mathbf{n}}\right)_s+\sigma(T_s-T_0), \tag{3}$$

where the subscript s reminds that this equation is applied to the grain surface only. The function J_s is an external energy flow onto the grain, which is caused by the recombination. The first term in the right-hand side of Eq. (3) represents the energy flow in the interior of the grain. We emphasize that the grain permanently interacts with the neutral gas, which is assumed to be uniform, with some temperature T_0 differing from the grain's surface temperature T_s. The interaction results in heat transfer, which is described by the second term in the right-hand side of Eq. (3). The rate of this natural cooling is characterized by the factor σ.

For a grain suspended above the electrode, the external energy flow may be taken in the form

$$J_s=\begin{cases} -\mathbf{J}_0\mathbf{n} & \text{for top half,} \\ 0 & \text{for bottom half.} \end{cases} \tag{4}$$

Equations (2)–(4) provide a complete description of the temperature distribution inside the grain. For a uniform spherical grain, they are readily solvable in terms of Legendre polynomials.

To proceed it is necessary to specify the collision of the neutral molecule with a grain surface. We undertake the assumption of complete energy accomodation. Since the size of a grain is usually small compared to the mean free path of the neutral molecule, the distribution function of the neutral gas in the vicinity of the grain can be taken as a combination of two Maxwellian distributions:

$$f(\mathbf{v})=\begin{cases} f_M(T_0) & \text{for } \mathbf{nv}<0, \\ f_M(T_s) & \text{for } \mathbf{nv}>0. \end{cases} \tag{5}$$

It should be noted that the number density of the neutral particles moving towards and outwards the grain (n_0 and n_s, respectively) is different:

$$n_0\sqrt{T_0}=n_s\sqrt{T_s} \tag{6}$$

in accordance with the conservation of the particle flux. Quantity n_0 is identical to the average number density of the neutral gas.

Using Eq. (5) one can evaluate the last term in the right-hand side of Eq. (3). By definition, the energy flux from the grain's surface, which is associated with the neutral gas and directed outward the grain, reads

$$\int\int\int \mathbf{n}\mathbf{v}\frac{mv^2}{2}f(\mathbf{v})d^3\mathbf{v}=\sqrt{\frac{2k^3}{\pi m}}(n_sT_s^{3/2}-n_0T_0^{3/2}), \tag{7}$$

where m is the mass of a neutral atom, k being the Boltzmann constant. By inserting n_s from Eq. (6) one can see that the right-hand side of Eq. (7) is of the form $\sigma(T_s-T_0)$ with

$$\sigma=n_0\sqrt{\frac{2k^3T_0}{\pi m}}.$$

In a similar way, one can evaluate the neutral gas pressure

$$P=\frac{1}{2}n_0k\sqrt{T_0}(\sqrt{T_0}+\sqrt{T_s}).$$

Note, that the pressure depends on the local surface temperature. Due to the inhomogeneous temperature distribution, the interaction with the neutral gas results in a force expressed as the integral

$$\mathbf{F}=\int\int(-P\mathbf{n})dS$$

over the grain surface.

In a case of a uniform spherical grain, the integration can be carried out analytically. The cumbersome final expression simplifies greatly in the ultimate case of $T_s-T_0\ll T_0$, which is the only one of interest here. The resulting force takes the form

$$F=\frac{\pi R^2n_0kJ_0}{6\left(\sigma+\frac{\varkappa}{R}\right)}, \tag{8}$$

where R is a radius of a grain.

Equation (8) closely resembles the known relation for the photophoretic force [7,8]. The difference is due to the different physical meaning of the term J_0. In the case of the photophoresis, the inhomogeneous heating is a result of external radiation. In a dusty plasma, the force is provided by the ion recombination.

In some experiments [9] the dusty specie is formed by relatively big hollow microspheres. Let the inner radius of a microsphere be ϵR with constant $\epsilon<1$. Assuming that the energy flux through the inner surface is negligibly small, one can easily obtain the following more general expression

$$F=\frac{1}{6}\frac{\pi R^2n_0kJ_0}{\sigma\left(1+\frac{1}{2}\epsilon^3\right)+\frac{\varkappa}{R}(1-\epsilon^3)}. \tag{9}$$

The relative contribution of the first term in the denominator of Eqs. (8) and (9) is characterized by dimensionless ratio

$$\frac{\sigma R}{\varkappa}=\frac{n_0R}{\varkappa}\sqrt{\frac{2k^3T_0}{\pi m}}.$$

For typical conditions of dust dynamics this ratio is negligible. Let's consider the argon gas with $n_0=10^{16}$ cm^{-3} and $T_0=300$ K°. Taking $\varkappa\approx10^5$ erg/sec·cm·K° (glasses) we estimate that $\sigma R/\varkappa=1$ for $R\approx3$ cm. Thus, under the laboratory conditions, the contribution of thermal conductivity dominates for all reasonable values of grain radius. For a hollow grain, say with $1-\epsilon^3=1/30$, the radius should be less than 0.1 cm. Then the final expression for the radiometric force reads

$$F=\frac{\pi R^3n_0kJ_0}{6\varkappa(1-\epsilon^3)}, \tag{10}$$

where J_0 is given in Eq. (1).

It should be noted that the ratio of this force to the gravity force is independent of the radius of the grain. Generally, it is extremely small. Let us compare the radiometric force with the ion-drag force, which is believed to play an important role under conditions of recent microgravity experiments with dusty plasmas [10].

The ion-drag force is estimated as $F_{\mathrm{drag}}=\alpha\pi R^2n_im_i(Mc_s)^2$, where the physical meaning of factor α is identical with that in Eq. (1). Hence

$$\frac{F}{F_{\mathrm{drag}}}=\frac{kRn_0Q}{6\varkappa m_iMc_s(1-\epsilon^3)}.$$

Note, that this ratio is independent on ion number density. To estimate the order of magnitude of the radiometric force, we assume that $M\approx1$, electron temperature $T_e=3\cdot10^4$ K°, then $c_s=2.5\cdot10^5$ cm/sec, $Q=15.8$ eV (argon) and $\varkappa$, n_0 and $1-\epsilon^3$ are the same as above. Then the discussed radiometric force is of the order of the ion-drag force, $F/F_{\mathrm{drag}}=1$, if $R=10^{-2}$ cm. The effect, therefore, should be observable for a dusty plasma composed of relatively large hollow grains under microgravity conditions.

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